Name: Feedback

SA367 · Mathematical Models for Decision Making — Spring 2022 · Uhan

Exam 2 – 3/31/2022

Instructions

- You have until the end of the class period to complete this exam.
- You may use your plebe-issue TI-36X Pro calculator.
- You may not use any other materials.
- **No collaboration allowed.** All work must be your own.
- **Show all your work.** To receive full credit, your solutions must be completely correct, sufficiently justified, and easy to follow.
- Keep this booklet intact.
- **Do not discuss the contents of this exam with any midshipmen until it is returned to you.**

Problem	Weight	Score
1	3	
2a	0.5	
2b	0.5	
3	3	
4a	0.5	
4b	0.5	
5a	1	
5b	1	
5c	0.5	
5d	1	
5e	0.5	
6a	1	
6b	1	
Total		/ 140

Problem 0. Copy and sign the honor statement below. This exam will not be graded without a signed honor statement.

> The Naval Service I am a part of is bound by honor and integrity. I will not compromise our values by giving or receiving unauthorized help on this exam.

Problem 1. You have been put in charge of managing the inventory of Tomahawk missiles at Navy Munitions Command Detachment Sewell's Point. Based on forecasts, you will need 3 missiles this month, then 2, 2, and 4 in successive months.

Missiles ordered in a given month arrive during the same month. Each month in which you place an order incurs a logistics and administration cost of \$200,000. Each missile costs \$700,000. Missiles can be held in inventory at a cost of \$100,000 per missile per month.

Regulations allow a maximum of 3 missiles to be ordered during each month. In addition, the size of the bunkers and other munitions restricts the ending inventory for each month to at most 2 missiles. At the beginning of this month, you have 1 missile in inventory.

Your goal is to find a plan that will meet all demands on time and minimizes the total ordering and holding costs over the next 4 months. Formulate this problem as a dynamic program by giving its shortest/longest path representation. In particular, follow the prompts below:

- Define the meaning of each stage.

- Define the meaning of the nodes in each stage.

- Draw the directed graph, specifying the nodes and edges. In one sentence, describe what the edges – not the edge lengths – represent. Do not specify the edge lengths.

If you struggled with this problem, see Example 3 and Problem 2 from Lesson 5 for similar problems. In particular, pay attention to how the states are defined.

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- Pick 3 edges in your graph, and specify their edge lengths. You may pick any 3 edges except those that involve the "end" node.

- Specify the source and sink nodes.

- Specify whether the goal is to find a shortest or longest path.

Problem 2. Suppose you solved the dynamic program you gave in Problem 1 by solving the corresponding shortest/longest path problem. The algorithm you used outputs (i) the length of a shortest/longest path and (ii) the nodes in a shortest/longest path.

For the questions below, you do not need to solve the dynamic program, just describe what you would do. Give a hypothetical example if it helps.

a. How would you use this output to determine the minimum total ordering and holding costs over the next 4 months?

b. How would you use this output to determine how many missiles to order in each month?

If you struggled with this problem, see Example 3 from Lesson 5 for a similar problem. In particular, for part b, make sure to explain precisely how you would determine how many missiles to order in each month.

Problem 3. Poisson Plastics has 13 production jobs it needs to process over the next 48 hours. The company has 3 identical machines that run in parallel. Each of these 13 jobs must be run on one of these machines nonpreemptively (i.e., once a job is started on a machine, it must stay on that machine until it is completed). Let p_t denote the processing time of job t, for $t = 1, \dots, 13$ in hours.

The company wants to minimize the makespan (i.e., the completion time of the last job to finish processing). Formulate this problem as a dynamic program by giving its shortest/longest path representation. In particular, follow the prompts below:

- Define the meaning of each stage.

- Define the meaning of the nodes in each stage.

- Sketch the edges from a node in stage t to all the relevant nodes in stage $t + 1$. Specify the edge lengths.

> If you struggled with this problem, see Example 2 from Lesson 8 for a similar problem. In particular, pay attention to how the states and edge lengths are defined.

- Sketch the edges from the last stage to the end node. Specify the edge lengths.

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- Specify the source and sink nodes.

- Specify whether the goal is to find a shortest or longest path.

Problem 4. Suppose you solved the dynamic program you gave in Problem 3 by solving the corresponding shortest/longest path problem. The algorithm you used outputs (i) the length of a shortest/longest path and (ii) the nodes in a shortest/longest path.

For the questions below, you do not need to solve the dynamic program, just describe what you would do. Give a hypothetical example if it helps.

a. How would you use this output to determine the minimum makespan?

b. How would you use this output to determine how to assign the jobs to the machines?

If you struggled with this problem, see Example 2 from Lesson 8 for a similar problem. In particular, for part a, pay attention to how the edge lengths are defined and how those contribute to the makespan of a schedule.

Problem 5. You have been hired as an analyst for Chauvenet Capital. You are in charge of deciding how to allocate \$10 million among 4 different investments:

	Cost (\$ millions)	Expected Return (\$100,000s)
Investment 1	6	8
Investment 2	3	5
Investment 3	4	6
Investment 4	2	4

In particular, you want to maximize the total expected return of the investments you choose. Assume that you cannot purchase fractional amounts of these investments.

Your predecessor started to formulate this problem as a dynamic program by giving its recursive representation:

- Let

$$c_t = \text{cost of investment } t \qquad r_t = \text{expected return of investment } t \qquad \text{for } t = 1, 2, 3, 4$$

- Stages:

$$\text{stage } t \leftrightarrow \begin{cases} \text{consider investment } t & \text{for } t = 1, 2, 3, 4 \\ \text{end of decision-making process} & \text{for } t = 5 \end{cases}$$

- States:

$$\text{state } n \leftrightarrow n \text{ million dollars remaining to invest} \quad \text{for } n = 0, \dots, 10$$

- Allowable decisions x_t at stage t and state n:

$$x_t \text{ must satisfy: } \quad \begin{array}{ll} (1) & x_t \in \{0, 1\} \\ (2) & c_t x_t \leq n \end{array} \quad \text{for } t = 1, \dots, 4 \text{ and } n = 0, \dots, 10$$

- Contribution of decision x_t at stage t and state n:

$$(3) \quad r_t x_t \qquad \text{for } t = 1, \dots, 4 \text{ and } n = 0, \dots, 10$$

- Value-to go function $f_t(n)$ at stage t and state n:

$$f_t(n) = \text{maximum total expected return with } n \text{ million dollars and investments } t, \dots, 4$$
$$\text{for } t = 1, \dots, 5 \text{ and } n = 0, \dots, 10$$

a. In words, briefly explain the allowable decisions described above.
In particular: what does $x_t = 0$ and $x_t = 1$ mean in constraint (1)? What does constraint (2) enforce?

Almost all of you had the right idea here. Make sure to explain what the allowable decisions $x_t = 0$ and $x_t = 1$ mean in the context of the problem.

b. In words, briefly explain why equation (3) above makes sense as the contribution of decision x_t at stage t and state n.

Almost all of you had the right idea here as well. Make sure to explain the contribution $r_t x_t$ using the correct interpretation of the allowable decisions.

Finish your predecessor's dynamic programming formulation by following the prompts below:

c. Specify the boundary conditions. <u>Make sure to specify which values of t or n apply: e.g., "for ..."</u>

This investment problem can be viewed as a knapsack problem. Take a look at Sections 0 and 1 of Lesson 11 for an example of formulating a knapsack problem as a dynamic program by giving its recursive representation.

d. Give the recursion on $f_t(n)$. <u>Make sure to specify which values of t or n apply: e.g., "for ..."</u>

e. Specify the desired value-to-go function value.

Problem 6. Consider the following dynamic programming recursion with stages $t = 1, 2, 3$ and states $n = 0, 1, 2$:

- Boundary conditions:
$$f_3(n) = 0 \quad \text{for } n = 0, 1, 2$$

- Recursion:
$$f_t(n) = \max_{\substack{x_t \in \{0,1\} \\ x_t \leq n}} \left\{ d_t x_t + f_{t+1}(n - x_t) \right\} \quad \text{for } t = 1, 2;\; n = 0, 1, 2$$

  where $d_1 = 2$ and $d_2 = 5$.

Below, the recursion is solved, but with some computations missing.

Stage 3:

$$f_3(n) = 0 \quad \text{for } n = 0, 1, 2$$

Stage 2:

$$\begin{aligned} f_2(2) &= \ldots \\ f_2(1) &= \max\{0 + f_3(1), 5 + f_3(0)\} = 5 \\ f_2(0) &= \max\{0 + f_3(0)\} = 0 \end{aligned}$$

Stage 1:

$$\begin{aligned} f_1(2) &= \max\{0 + f_2(2), 2 + f_2(1)\} = 7 \\ f_1(1) &= \max\{0 + f_2(1), 2 + f_2(0)\} = 5 \\ f_1(0) &= \ldots \end{aligned}$$

a. Fill in the missing computations for $f_2(2)$ and $f_1(0)$.

b. Suppose the desired value-to-go value is $f_1(1) = 5$. What is the corresponding optimal solution? In other words, what are the values of x_1 and x_2 that achieve this value of $f_1(1)$?

If you struggled with this problem, take a look at the examples of solving DP recursions in Lesson 11, as well as the homework problems in that lesson. In particular, for part a, make sure that you correctly identify the allowable decisions (e.g. is x_t allowed to be 0? 1?). For part b, make sure that you correctly identify which allowable decision gives you the maximum value for each value-to-go function value.